CALCULATION OF ANGULAR IRRADIATION COEFFICIENTS FOR A THREE-DIMENSIONAL MULTICOUPLED REGION

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An algorithm is presented for calculating angular coefficients for isothermal zones of complex shape in a three-dimensional region involving shadowing.

It has now become common design practice to compute the temperature distributions in instrument racks, and a considerable part is played in the heat balance in such cases by radiation, which requires a computer algorithm to define the mutual irradiation coefficients for the components of a three-dimensional multicoupled object of complex shape (as far as possible without restriction), which must be universal, i.e., applicable to a wide range of objects.

At present, these coefficients involving bodies with partial shadowing are determined by experiment [1] or by Monte Carlo methods [2], although in both cases the accuracy is restricted. Here we present a numerical method and program for calculating the coefficients for bodies in a three-dimensional region, whose accuracy is adequate for engineering purposes.

We approximate the actual surfaces within and on an instrument section (zones) as areas bounded by convex piecewise-linear surfaces. The division into zones is made in such a way that the entire surface of each zone may be considered as isothermal and diffusely emitting, namely, gray with a constant degree of blackness. This approximation is adequate for engineering purposes and allows one to represent virtually any object orovided that the number of zones is large enough. Of course, restriction on the store volume may prevent one from using a very large number of zones, but zones distinguished solely from geometrical considerations may often be combined into piecewise-planar isothermal zones if there are no marked differences in temperature and blackness, with the result that the total number of zones involved in the thermal calculation proper may not be very large.

The basic purpose of the calculation is to determine the matrix

$$\varphi_t = \begin{vmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1N} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ \varphi_{N1} & \varphi_{N2} & \cdots & \varphi_{NN} \end{vmatrix}, \tag{1}$$

where N is the number of isothermal zones; for each term in (1) we can write that

$$\varphi_{t1,t2} = \frac{1}{F_{t1}} \int_{F_{t_1}} dF_{t1} \int_{F_{t_2}} d\varphi_{t1,t2}. \tag{2}$$

This model (a set of planar zones) allows (2) to be replaced by the finite sum

$$\varphi_{t1,t2} = \frac{1}{F_{t1}} \sum_{i=1}^{nt1} F_i \sum_{j=1}^{nt2} \varphi_{ij}, \tag{3}$$

where nt_1 and nt_2 are the numbers of planar zones that simulate the isothermal zones t_1 and t_2, while

$$\varphi_{ij} = \frac{1}{F_i} \int_{F_i} \int_{F_j} \frac{\cos\varphi_i \cos\varphi_j dF_i dF_j}{\pi r_{ij}^2}, \tag{4}$$

where φ_i and φ_j are the angles between the line joining any two points directly visible one from the other (unshadowed) and the normals to these.

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TABLE 1. Angular Coefficients for Surfaces in a Container Enclosing Three Units

i2	i1											Σ
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	
I	0	0,0387	0	0,4649	0,0766	0,0022	0,2514	0,0225	0,0003	0,1421	0,0003	0,9989
II	0,0387	0	0,0387	0,0766	0,4649	0,0771	0,0225	0,2514	0,0237	0,0021	0,0021	0,9984
III	0	0,0387	0	0,0022	0,0766	0,4940	0,0003	0,0225	0,2632	0,0003	0,1006	0,9984
IV	0,3229	0,0532	0,0015	0,2299	0,0570	0,0037	0,1526	0,0324	0,0015	0,1490	0,0016	1,0052
V	0,0532	0,3229	0,0532	0,0570	0,2299	0,0582	0,0324	0,1526	0,0327	0,0080	0,0071	1,0072
VI	0,0014	0,0471	0,3020	0,0032	0,0512	0,2504	0,0013	0,0284	0,1691	0,0017	0,1490	1,0048
VII	0,4095	0,0367	0,0004	0,3578	0,0759	0,0035	0	0	0	0,1227	0,0010	1,0076
VIII	0,0367	0,4095	0,0367	0,0759	0,3578	0,0758	0	0	0	0,0053	0,0042	1,0020
IX	0,0004	0,0339	0,3774	0,0031	0,0675	0,3965	0	0	0	0,0011	0,1266	1,0066
X	0,3134	0,0063	0,0007	0,4730	0,0255	0,0063	0,1661	0,0071	0,0017	0	0,0065	1,0065
XI	0,0006	0,0045	0,2218	0,0050	0,0227	0,5375	0,0014	0,0057	0,1948	0,0065	0	1,0005

The integral (4) can be converted to a curvilinear integral over the boundaries of the unshadowed zones:

$$\varphi_{ij} = \frac{1}{2\pi F_i} \oint_{C_i} \oint_{C_j} \ln r_{ij} (dx_i dx_j + dy_i dy_j + dz_i dz_j). \tag{5}$$

The visibility condition for any two points in the zones is put as

$$\begin{gathered} \Delta x_{ij} \cos \alpha_i + \Delta y_{ij} \cos \beta_i + \Delta z_{ij} \cos \gamma_i > 0, \\ -\Delta x_{ij} \cos \alpha_j - \Delta y_{ij} \cos \beta_j - \Delta z_{ij} \cos \gamma_j > 0. \end{gathered} \tag{6}$$

The φ_{ij} are calculated by first analyzing whether zone i is visible from zone j; this will not be so if

$$\cos \varphi_i \cos \varphi_j = 1 \vee (\cos \varphi_i \cos \varphi_j = -1 \wedge p_i + p_j = 0). \tag{7}$$

Zones i and j are visible directly from one another if: 1) They are not obscured by other zones; 2) the lines of intersection of the planes lie outside the edges of the zones; and 3) any two points on the zones are visible one from the other, namely (6), applies.

Zones i and j are partially visible from one another if the edge of one of them intersects the plane of the other; then the convexity of the edge means that the intersected zone is divided into not more than two parts. In that case, the program replaces the given edge of the intersected zone by the part visible from the other zone.

The numbers of the zones that obscure zones i and j are determined by testing the ray paths, namely, the part of space containing all lines between the sets of points in zones i and j. One examines only rays joining all the vertices and the centers of gravity of the two zones. Some other zone (a zone not coincident with i or j) is an obscuring zone if a ray passes through it.

After the latter test, the coefficient is calculated by numerical integration of (5) if there is no shadowing or from (4) if zones i and j are partially obscured.

The edges c_i and c_j are piecewise-linear, so the integral of (5) can be replaced by the following sum of integrals:

$$\varphi_{ij} = \frac{1}{2\pi F_i} \sum_{ii=1}^{nwi} \sum_{jj=1}^{nwj} \int_{a_{ii}}^{b_{ii}} \int_{a_{jj}}^{b_{jj}} \ln r_{ij} (dx_{ii} dx_{jj} + dy_{ii} dy_{jj} + dz_{ii} dz_{jj}), \tag{8}$$

where nwi and nwj are the numbers of sides on the edges of zones i and j; and a_{ii}, b_{ii}, a_{jj}, b_{jj} are the limits of integration along the sides ii and jj of zones i and j; the inner integrals in (8) are calculated by Simpson's method.

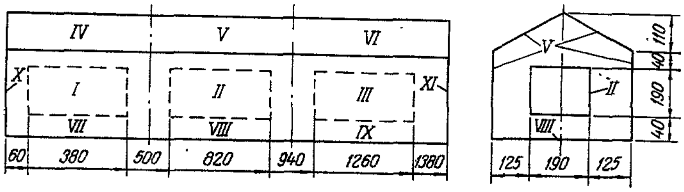

Fig. 1. Isothermal surfaces I-XI in a container and in the units.

If there is partial shadowing, there are regions of discontinuity over the areas F_i and F_j in the integrand in (4), which means that the integral of (5) cannot be used; the integral of (4) is replaced by

$$\varphi_{ij_{eh}} = \frac{1}{2\pi F_i} \sum_{ki=1}^{ni} \sum_{kj=1}^{nj(ki)} \frac{\cos\varphi_{ki}\cos\varphi_{kj}}{r^2_{ki,kj}} \Delta F_i \Delta F_j, \tag{9}$$

where ni and nj are the numbers of elementary areas in zones i and j visible from one another.

Replacement of (4) by (9) results in an error that decreases as we increase the number of subzones in zones i and j, i.e., we perform successive approximations and calculate φ_{ij} by increasing ni and nj successively until the following condition is met:

$$\text{abs}\left(\frac{\varphi^{it+1}_{ij_{eh}} - \varphi^{it}_{ij_{eh}}}{\varphi^{it+1}_{ij_{eh}}}\right) \leqslant \text{eps } 1, \tag{10}$$

where it is the number of the approximation and eps 1 is the acceptable error.

Two types of decomposition into elementary areas are used in the integration; any zone whose principal projection is a rectangle is split up via a rectangular grid into equal elements approximating to squares. A polygon is divided up by its diagonals originating from the vertex of maximum internal angle to give triangles, each of which is itself split up by a network of lines parallel to the sides to give equal triangles similar to the initial one.

The geometrical parameter that defines the dimensions of the initial decomposition is the dimension dl representing the diameter of the average element for the region. The number of divisions for a triangular subregion or for the shorter side of a rectangular region is increased by one on going to the next approximation. The iteration is terminated if (10) is met or if the number of divisions exceeds the acceptable number max n.

The calculation utilizes the reciprocity relation

$$\varphi_{t1,t2}F_{t1} = \varphi_{t2,t1}F_{t2}, \tag{11}$$

which means that only half of the matrix of (1) need be calculated; the correctness is tested by means of a closure condition implied by the conservation of energy:

$$\sum_{t1=1}^{N} \varphi_{t1,t2} = 1. \tag{12}$$

The absolute error in calculating a single row in (1) is indicated by

$$\Delta\xi = \text{abs}\left(1 - \sum_{t1=1}^{N} \varphi_{t1,t2}\right). \tag{13}$$

This algorithm has been used in an ALGOL-60 program for the BÉSM-6 computer; the input data consist of lists of numbers of the planar zones constituting the isothermal surfaces, lists of the numbers of the points describing the edges of the zones in the counterclockwise sense (from the end of the normal), and the array of coordinates for the zone vertices.

The performance is illustrated by results for the coefficients within a container enclosing three units; the internal surface of the container and the surfaces of the units were split up into isothermal surfaces as shown in Fig. 1, while Table 1 gives the results as the matrix of the angular coefficients.

NOTATION

0xyz	is the orthogonal coordinate system;
φ_{ij}	is the generalized mean coefficient for the irradiation of the i-th zone by the j-th zone;
F_i	is the zone area;
$\alpha_i, \beta_i, \gamma_i$	are the direction angles of $\bar{n}_i$, the normal to F_i;
$abs(p_i)$	is the length of radius vector p_i for zone i;
c_i	is the contour of the i-th zone;
r_{ij}	is the distance between points in the i-th and j-th zones;
$\Delta x_{ij}, \Delta y_j, \Delta z_{ij}$	are the projections of r_{ij} on the x, y, and z axes.

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THERMAL CONDUCTIVITY OF A REINFORCED PLATE

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Multiscale expansion is used in asymptotic integration of a steady-state heat-conduction problem for a thin plate of periodic structure; results are presented for the boundary layer near the end.

The singular-perturbation method has proved an efficient means of derving approximate equations for thin bodies; for instance, the method has been applied to the complete equations in the theory of elasticity to derive equations for the bending of a plate [1, 2] or rod [3]. A similar method has been used [4] in the theory of heat conduction for a thermally insulated lateral surface. A method has been given [5, 6] for extending the technique to conditions of the third kind for small values of the Biot number. An approximation has also been constructed [7] for the asymptote to a second boundary-value problem for a second-order elliptic equation of general form for a region in which one dimension is much less than the others.

These studies have envisaged either homogeneous bodies or else bodies in which the parameters vary slowly in space; on the other hand, applications often involve inhomogeneous media in which the parameters vary considerably over distances small by comparison with the length of the body. The simplest case is one where the rapid change is regular, e.g., periodic. Bodies of regular structure are of importance in themselves in the description of reinforced structures [8] as well as in the simulation of irregular inhomogeneous bodies, including random media. The asymptotic methods of [1-3, 5-7] are inadequate for media with rapidly varying parameters. However, another form of the singular-perturbation method, which is widely used in nonlinear mechanics [4], is then effective: two-scale expansion. Here we consider a steady-state problem in the theory of heat conduction for a thin plate reinforced by a rod lattice. It is assumed that the thermal conductivity of the reinforcement differs from that of the matrix material and also that the thermal contact is ideal.

The latter assumption is unimportant for the method given here and is made only in order to simplify the expressions.

Physical considerations show that such a reinforced plate can be replaced approximately by a homogeneous plate whose thermal conductivity along the rod direction is different from that along the transverse direction if we are not interested in the details of the temperature variation over distances small by comparison with the size of the plate in plan. Here we provide a justification for this substitution, i.e., we use the three-dimensional conduction equation to derive a two-dimensional one and construct an algorithm for calculating the corrections to the two-dimensional temperature distribution. This method gives, in particular,

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